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The $U_L(3) \times U_R(3)$ Extended Nambu–Jona-Lasinio Model in Differential Regularization

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ABSTRACT

We employ the method of differential regularization to calculate explicitly the one-loop effective action of a bosonized $U_L(3) \times U_R(3)$ extended Nambu–Jona-Lasinio model consisting of scalar, pseudoscalar, vector and axial vector fields.

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1. Introduction

Quantum chromodynamics (QCD) is a theory of strong interactions [1] and is asymptotically free such that the forces between quarks become weak for small quark-quark separation or, equivalently, large momentum transfer. The same self-interactions of gluons that give rise to asymptotic freedom lead to a strong quark-quark interaction for medium and small energies, and thus the low energy hadron physics cannot be handled by perturbative QCD. At low energies, chiral perturbation theory [2] is a good method for perturbative calculations. We note that a disadvantage of chiral perturbation theory is that as soon as we go beyond the lowest order, the number of free parameters increases very rapidly, thus making calculations beyond the lowest few orders rather impractical. We would thus like to obtain these free parameters directly from QCD. This is rather difficult to do so far, and there is a need for some models that interpolate between QCD and chiral perturbation theory.

Among many models, the Nambu–Jona-Lasinio (NJL) model [3,4] seems to be the simplest pure quark theory, which yields dynamical symmetry breaking and hence a nonvanishing value for the quark condensate. But to make the perturbation series nondivergent, we need to introduce an ultraviolet cutoff as a regularization. Many regularization methods have been employed [5] and it was found that the one-loop effective potentials in various

methods can vary substantially with respect to the change of the renormalization scale. In particular, one should know that not all regularization methods are suitable to NJL models. Moreover, higher loop corrections are also needed to reduce the sensitivity of the renormalization scheme dependence of perturbative results.

In this paper, we shall use a space-time regularization method known as differential regularization (DR) [6,7] to calculate the one-loop effective action of an extended NJL (ENJL) model. This method has been shown to be useful for studying the quantum corrections in chiral theories. By using DR method, we can determine the effective action systematically and unambiguously. We organize the paper as follows. In section 2, we introduce the ENJL model and the calculational procedure of DR method. In section 3, we calculate explicitly the one-loop effective action of the ENJL model.

2. The Extended NJL Model and Differential Regularization

We consider the $U_L(3) \times U_R(3)$ extended NJL Lagrangian for quark field [8,9]:

$$\begin{aligned} \mathcal{L}_{ENJL} &= \bar{q}(i\gamma^\mu \partial_\mu - m_0)q \\ &\quad +2G_1 \sum_{i=0}^{N_f^2-1} \left[(\bar{q}\frac{\lambda^i}{2}q)^2 + (\bar{q}\frac{\lambda^i}{2}i\gamma_5 q)^2 \right] \end{aligned}$$

$$-2G_2\sum_{i=0}^{N_f^2-1}\left[(\bar{q}\frac{\lambda^i}{2}\gamma_\mu q)^2+(\bar{q}\frac{\lambda^i}{2}i\gamma_5\gamma_\mu q)^2\right], \tag{1}$$

where G_1 and G_2 are the four-fermion coupling constants, m_0 is the quark mass matrix and λ^i are Gell-Mann matrices.

The quantized theory can be written in terms of a generating functional which, in the absence of external sources, reads

$$Z_{ENJL}=\int D\bar{q}Dq\exp\left[i\int d^4x\mathcal{L}_{ENJL}(x)\right]. \tag{2}$$

In order to use the model to describe the low energy properties with the manifest low energy modes, we bosonize the model with the auxiliary fields $\Phi=\Phi_\alpha\Lambda_\alpha$ introduced via the identity

$$\begin{aligned}&\exp\left(-\frac{i}{2}\int\bar{q}\Lambda_\alpha qQ^{\alpha\beta}\bar{q}\Lambda_\beta q\right)\\&\qquad=\int D\Phi\exp\left(-\frac{i}{2}\int\Phi_\alpha(Q^{-1})^{\alpha\beta}\Phi_\beta-i\int\Phi_\alpha\bar{q}\Lambda_\alpha q\right).\end{aligned} \tag{3}$$

It contains therefore (in the case of three flavors) nonets of scalar, pseudoscalar, vector and axial vector meson fields:

$$\Lambda_\alpha=\frac{\lambda^i}{2}\otimes\Gamma_a,\quad i=0,\cdots,N_f^2-1,\quad\Gamma_a\in\{1,\quad i\gamma_5,\quad i\gamma_\mu,\quad i\gamma_\mu\gamma_5\}, \tag{4}$$

$$Q^{\alpha\beta}=\begin{cases}4G_1\delta^{\alpha\beta} & \text{for}\quad\Gamma_a\in\{1,\quad i\gamma_5\}\\4G_2\delta^{\alpha\beta} & \text{for}\quad\Gamma_a\in\{\gamma_\mu,\quad\gamma_\mu\gamma_5\}\end{cases}. \tag{5}$$

Hence we have

$$\mathcal{Z}_{ENJL}=\int D\Phi\exp\left(-\frac{i}{2}\int\Phi Q^{-1}\Phi\right)Z_F[\Phi], \tag{6}$$

where

$$Z_F[\Phi] = \int DqD\bar{q} \exp\left(-i \int \bar{q}(i\gamma^\mu \partial_\mu - m_0 - \Phi)q\right) . \tag{7}$$

By shifting $\Phi \to \Phi - m_0$, we have

$$\mathcal{Z}_{ENJL} = \int D\Phi e^{-\frac{i}{2}\int d^4x(\Phi - m_0)Q^{-1}(\Phi - m_0)} \int DqD\bar{q} e^{-i\int d^4x \bar{q}(i\gamma^\mu\partial_\mu - \Phi)q} . \tag{8}$$

The auxiliary field Φ can be expressed in terms of the scalar, pseudoscalar, vector and axial vector fields as

$$\Phi = g_S S + ig_P\gamma_5 P - ig_V\gamma^\mu V_\mu - ig_A\gamma^\mu A_\mu\gamma_5 . \tag{9}$$

Then we can write

$$\begin{aligned} \frac{1}{2}(\Phi - m_0)Q^{-1}(\Phi - m_0) &= \frac{1}{2G_1} Tr((S - m_0)^2 + P^2) \\ &\quad + \frac{1}{2G_2} Tr(V_\mu V^\mu + A_\mu A^\mu) , \end{aligned} \tag{10}$$

where $S = \sum_{i=0}^{8} \frac{\lambda^i}{2} S^i$, $P = \sum_{i=0}^{8} \frac{\lambda^i}{2} P^i$, $V_\mu = \frac{\lambda^0}{2}\omega_\mu + \sum_{i=1}^{8} \frac{\lambda^i}{2}\rho^i_\mu$, and $A_\mu = \frac{\lambda^0}{2} f_\mu + \sum_{i=1}^{8} \frac{\lambda^i}{2} a^i_\mu$.

By including the electroweak interaction, we have the generating functional

$$\begin{aligned} \mathcal{Z}_{ENJL} &= \int DWDBD\Phi e^{-\int d^4x \frac{i}{2}(\Phi - m_0)Q^{-1}(\Phi - m_0) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}} \\ &\quad \times \int DqD\bar{q} e^{-i\int d^4x \bar{q}(i\gamma^\mu\partial_\mu - \Phi)q} , \end{aligned} \tag{11}$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu , \\ G_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu + g_W [W_\mu, W_\nu] . \end{aligned} \tag{12}$$

Then the auxiliary field Φ is expressed as the scalar, pseudoscalar, vector, axial vector fields and gauge bosons,

$$\Phi = g_S S + ig_P\gamma_5 P - ig_V\gamma^\mu V_\mu - ig_A\gamma^\mu A_\mu\gamma_5 - ig_W\gamma^\mu W_\mu - ig_B\gamma^\mu B_\mu\,, \quad (13)$$

where W and B are the electroweak gauge bosons. The fields S and P can be represented chirally as

$$S + i\gamma_5 P = P_R M + P_L M^\dagger\,, \quad (14)$$

where $P_R = \frac{1}{2}(1+\gamma_5)$ and $P_L = \frac{1}{2}(1-\gamma_5)$. We can parameterize $M = S + iP$ as $M = U\Sigma U$, with Σ being the scalar σ fields, and U the coset $U_L(3)\times U_R(3)/U_V(3)$:

$$\begin{aligned} U &= \exp\left\{ i\sqrt{2}\frac{\Phi_8+\Phi_1}{f_0}\right\}\,, && (15)\\ \Phi_8 &= \sum_{i=1}^{8}\frac{\lambda^i}{\sqrt{2}}\phi^i \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & k^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & k^0 \\ k^- & \bar{k}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}\,, && (16)\\ \Phi_1 &= \frac{1}{\sqrt{3}}\eta_1 I\,. && (17) \end{aligned}$$

In the low-energy approximation, we couple the scalar, pseudoscalar, vector and axial vector fields to external sources. The quark fields are not coupled to any external sources and can be integrated over to give a functional determinant, whose evaluation requires a regularization. The calculation can be carried out by using the Feynman rules for the quark fields, such that the

one-loop quantum effects are considered by treating quarks as internal lines in the Feynman diagrams.

In this model, it is necessary to determine its vacuum by calculating the effective potential or action to at least one-loop order. We shall employ differential regularization and carry out the calculation in Euclidean space, so that the functional integral reads

$$Z'_{ENJL} = \int D\Phi \exp\left[-\mathcal{S}_{eff}(\Phi)\right] , \tag{18}$$

where the effective action in the one-loop fermion approximation is

$$\begin{aligned} \mathcal{S}_{eff} \quad &= \quad -\ln \mathrm{Det}[-i\gamma^{\mu}\partial_{\mu} + \Phi] \\ &\quad +\frac{1}{2}\int d^4x(\Phi - m_0)Q^{-1}(\Phi - m_0) \, . \end{aligned} \tag{19}$$

The functional determinant can be calculated using the Feynman rules; in particular, the massless quark propagator is given by

$$\begin{aligned} \langle q^i_a(x)\bar{q}^j_b(0)\rangle \quad &\equiv \quad \Delta^{ij}_{ab}(x) \\ &= \quad -\frac{i\delta^{ij}\delta_{ab}}{2\pi^2}\frac{\gamma_{\mu}x^{\mu}}{x^4} \, , \end{aligned} \tag{20}$$

with i, j and a, b being the isospin and color indices, respectively.

In perturbative calculations, we encounter highly singular terms of the form

$$\frac{1}{(x^2)^n}\ln^m(\mu^2 x^2), \quad n \geq 2 \, , m \geq 0 \, , \tag{21}$$

where μ is a mass parameter in the problem. The essential idea of differential regularization is to define these highly singular terms by

$$\frac{1}{(x^2)^n}\ln^m(\mu^2 x^2) \equiv \underbrace{\Box\Box\ldots\Box}_{n-1} G(x^2)\,,\, x^2 \neq 0\,, \tag{22}$$

where $G(x^2)$ is a to-be-determined function that has a well-defined Fourier transform and can depend on $2(n-1)$ integration constants, which play the role of a subtraction scale. In this paper, we encounter only the following two forms:

$$\frac{1}{(x^2)^2} \to -\frac{1}{4}\Box\frac{\ln x^2\mu^2}{x^2}\,,\, x^2 \neq 0\,, \tag{23}$$

$$\frac{1}{(x^2)^3} \to -\frac{1}{32}\Box\Box\frac{\ln x^2\mu^2}{x^2}\,,\, x^2 \neq 0\,, \tag{24}$$

where the mass parameter μ is an integration constant. Note that we have omitted other irrelevant integration constants for $x^2 \neq 0$. This regularization method has been used in ϕ^4 theory, QCD [6] and Nambu–Jona-Lasinio model [7] and can reproduce the well-known results obtained by other methods. The advantage of DR method is that loop corrections in a chiral theory can be calculated unambiguously. We note also that different methods can lead to different results for the one-loop effective potential, which depends strongly on renormalization scheme. Higher loop corrections can reduce the sensitivity of scheme dependence, but of course, the use of a regularization method requires special attention.

3. The One-Loop Effective Action

To obtain the one-loop effective action of the ENJL model, we need to evaluate the one-loop bilinear terms in the scalar, pseudoscalar, vector and axial vector fields, and their interaction terms. For illustrative purposes, we evaluate some of them below. The bilinear term in the scalar field with an internal quark loop is easily calculated and reads

$$\begin{aligned}
\Pi_S(x,y) &= -\frac{g_S^2}{2}\int d^4xd^4y\mathrm{Tr}\left[\Delta(x-y)\Delta(y-x)S(x)S(y)\right] \\
&= -\frac{3g_S^2}{2\pi^4}\int d^4xd^4y\mathrm{Tr}\left[S(x)S(y)\right]\frac{1}{(x-y)^6} \\
&\overset{sing.}{=} \frac{3g_S^2}{128\pi^4}\int d^4xd^4y\left[\frac{3}{2}S^0(x)S^0(y)+\sum_{i=1}^{8}S^i(x)S^i(y)\right] \\
&\quad\times\Box\Box\frac{\ln(x-y)^2\mu^2}{(x-y)^2}\,,(x-y)^2\neq 0\,,
\end{aligned} \tag{25}$$

where $\Box=\Box_{(x-y)}$ and we have used $\mathrm{Tr}_\gamma I=4,\mathrm{Tr}_c I=3$, and $\mathrm{Tr}_\lambda I=3$ for the spinor, color, and octet degrees of freedom, respectively. The logarithmic dependence of the scalar term reads

$$\frac{d\Pi_S}{d\ln\mu^2}=-\frac{3g_S^2}{32\pi^2}\int d^4x\left[\frac{3}{2}S^0(x)\Box S^0(x)+\sum_{i=1}^{8}S^i(x)\Box S^i(x)\right]\,. \tag{26}$$

The bilinear term in the pseudoscalar field is

$$\begin{aligned}
\Pi_P(x,y) &= -\frac{g_P^2}{2}\int d^4xd^4y\mathrm{Tr}\left[i\gamma_5\Delta(x-y)i\gamma_5\Delta(y-x)P(x)P(y)\right] \\
&= -\frac{3g_P^2}{2\pi^4}\int d^4xd^4y\mathrm{Tr}\left[P(x)P(y)\right]\frac{1}{(x-y)^6} \\
&\overset{sing.}{=} \frac{3g_P^2}{128\pi^4}\int d^4xd^4y\left[\frac{3}{2}P^0(x)P^0(y)+\sum_{i=1}^{8}P^i(x)P^i(y)\right] \\
&\quad\times\Box\Box\frac{\ln(x-y)^2\mu^2}{(x-y)^2}\,,
\end{aligned} \tag{27}$$

and its logarithmic dependence is

$$\frac{d\Pi_P}{d\ln\mu^2} = -\frac{3g_P^2}{32\pi^2}\int d^4x \left[\frac{3}{2}P^0(x)\Box P^0(x) + \sum_{i=1}^{8} P^i(x)\Box P^i(x)\right] . \qquad (28)$$

The bilinear term in the vector field is

$$\begin{aligned}
\Pi_V(x,y) \quad &= \quad -\frac{g_V^2}{2}\int d^4x d^4y \mathrm{Tr}\left[i \not{V}(x)\Delta(x-y) i \not{V}(y)\Delta(y-x)\right] \\
&= \quad \frac{3g_V^2}{2\pi^4}\int d^4x d^4y \mathrm{Tr}\left[(\frac{\lambda^i}{2}\rho^i_\alpha(x) + \frac{\lambda^0}{2}\omega_\alpha(x))(\frac{\lambda^j}{2}\rho^j_\beta(y) + \frac{\lambda^0}{2}\omega_\beta(y))\right] \\
&\quad \times\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^8}\mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu) \\
&\stackrel{sing.}{=} \quad \frac{3g_V^2}{32\pi^4}\int d^4x d^4y \left[\frac{3}{2}\omega^\mu(x)\omega_\mu(y) + \sum_{i=1}^{8}\rho^{i\mu}(x)\rho^i_\mu(y)\right] \\
&\qquad\qquad \times \Box\Box\frac{\ln(x-y)^2\mu^2}{(x-y)^2} . \qquad (29)
\end{aligned}$$

Its logarithmic dependence is

$$\frac{d\Pi_V}{d\ln\mu^2} = -\frac{3g_V^2}{8\pi^2}\int d^4x \left[\frac{3}{2}\omega^\mu(x)\Box\omega_\mu(x) + \sum_{i=1}^{8}\rho^{i\mu}(x)\Box\rho^i_\mu(x)\right] . \qquad (30)$$

The bilinear term in the axial vector field is

$$\begin{aligned}
\Pi_A(x,y) \quad &\stackrel{sing.}{=} \quad \frac{3g_A^2}{32\pi^4}\int d^4x d^4y \left[\frac{3}{2}f^\mu(x)f_\mu(y) + \sum_{i=1}^{8}a^{i\mu}(x)a^i_\mu(y)\right] \\
&\qquad\qquad \times \Box\Box\frac{\ln(x-y)^2\mu^2}{(x-y)^2} , \qquad (31)
\end{aligned}$$

and its logarithmic dependence is

$$\frac{d\Pi_A}{d\ln\mu^2} = -\frac{3g_A^2}{8\pi^2}\int d^4x \left[\frac{3}{2}f^\mu(x)\Box f_\mu(x) + \sum_{i=1}^{8}a^{i\mu}(x)\Box a^i_\mu(x)\right] . \qquad (32)$$

The one-loop vector-scalar-scalar interaction reads

$$
\begin{aligned}
\Gamma_{VSS}(x,y,z) &= -g_V g_S^2 \int d^4x d^4y d^4z \mathrm{Tr}\left[-i \not{V}(x)\Delta(x-y)S(y)\right. \\
&\quad \left.\times\Delta(y-z)S(z)\Delta(z-x)\right] + (\text{permut.}) \\
&= -\frac{3g_V g_S^2}{32\pi^6}\int d^4x d^4y d^4z \mathrm{Tr}\left[\left(\frac{\lambda^i}{2}\rho^i_\mu(x) + \frac{\lambda^0}{2}\omega_\mu(x)\right)\frac{\lambda^j}{2}S^j(y)\right. \\
&\quad \left.\times\frac{\lambda^k}{2}S^k(z)\right] \mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\gamma\right] \frac{(x-y)_\alpha(y-z)_\beta(z-x)_\gamma}{(x-y)^4(y-z)^4(z-x)^4} \\
&\overset{sing.}{=} \frac{3g_V g_S^2}{128\pi^4}\int d^4x d^4y d^4z (\omega^\mu(x)\partial_\mu S^i(y)S^i(z) \\
&\quad +(if_{abc}+d_{abc})\rho^{a\mu}(x)\partial_\mu S^b(y)S^c(z)) \\
&\quad \times\left[3\delta(x-z)\Box\frac{\ln(x-y)^2\mu^2}{(x-y)^2}\right. \\
&\qquad \left. +\delta(x-z)\Box\frac{\ln(y-z)^2\mu^2}{(y-z)^2}\right], \qquad (33)
\end{aligned}
$$

and the logarithmic dependence is

$$
\begin{aligned}
\frac{d\Gamma_{VSS}}{d\ln\mu^2} &= -\frac{g_V g_S^2}{128\pi^4}\int d^4x d^4y d^4z (\omega^\mu(x)\partial_\mu S^i(y)S^i(z) \\
&\quad +(if_{abc}+d_{abc})\rho^{a\mu}(x)\partial_\mu S^b(y)S^c(z))\delta(x-z) \\
&\quad \times\left[12\pi^2\delta(x-y) + 4\pi^2\delta(y-z)\right] \\
&= -\frac{g_V g_S^2}{8\pi^2}\int d^4x (\omega^\mu(x)\partial_\mu S^i(x)S^i(x) \\
&\quad +(if_{abc}+d_{abc})\rho^{a\mu}(x)\partial_\mu S^b(x)S^c(x)) . \qquad (34)
\end{aligned}
$$

Other one-loop quark contributions are listed in the appendix.

4. Discussion

We have calculated the one-loop effective action of the ENJL model by the method of differential renormalization, which seems to be a very simple method. We note that γ_5 is not well defined in arbitrary dimensions of space-time. But differential renormalization performs loop calculation in coordinate space with well defined γ_5. Indeed, the bosonized ENJL model gives the low energy meson effective Lagrangian by means of quark loop contributions. It is a model that interpolates between QCD and chiral perturbation theory. The original ENJL Lagrangian which has four-fermion interaction is non-renormalizable, but the bosonized version is renormalizable. The anomalous part of the effective action can be calculated by the WZW method and higher loop can be considered further.

Appendix

Here we list the rest of the one-loop logarithmic quark contributions to the effective action.

$$\begin{aligned} \frac{d\Gamma_{VPP}}{d\ln\mu^2} &= \frac{g_V g_S^2}{8\pi^2}\int d^4x \Big[\omega^\mu(\partial_\mu P^i)P^i \\ &\quad +(if_{abc}+d_{abc})\rho^{a\mu}(\partial_\mu P^b)P^c\Big]\,, \end{aligned} \tag{35}$$

$$\Gamma_{VPS}(x,y,z)=\Gamma_{ASS}(x,y,z)=\Gamma_{APP}(x,y,z)=0\,, \tag{36}$$

$$\begin{aligned} \frac{d\Gamma_{APS}}{d\ln\mu^2} &= \frac{g_A g_S g_P}{4\pi^2}\int d^4x \Big[(\partial^\mu f_\mu)S^iP^i \\ &\quad +(if_{abc}+d_{abc})(\partial^\mu a^a_\mu)S^bP^c\Big]\,, \end{aligned} \tag{37}$$

$$
\begin{aligned}
\frac{d\Gamma_{AAV}}{d\ln\mu^2} &= \frac{g_V g_A^2}{8\pi^2}\int d^4x\left\{\frac{7}{2}a^{a\mu}(\partial^\nu\rho^a_\nu)f_\mu - 4a^{a\mu}\rho^a_\mu\partial^\nu f_\nu\right. \\
&\quad -\frac{1}{2}a^{a\mu}(\partial^\nu\rho^a_\mu)f_\nu - \frac{1}{2}a^{a\mu}(\partial_\mu\rho^{a\nu})f_\nu + \frac{7}{2}a^{a\mu}(\partial^\nu\omega_\nu)a^a_\mu \\
&\quad -4a^{a\mu}\omega_\mu\partial^\nu a^a_\nu - \frac{1}{2}a^{a\mu}(\partial^\nu\omega_\mu)a^a_\nu - \frac{1}{2}a^{a\mu}(\partial_\mu\omega_\nu)a^a_\nu \\
&\quad +\frac{7}{2}f^\mu(\partial^\nu\rho^a_\nu)a^a_\mu - 4f^\mu\rho^a_\mu\partial^\nu a^a_\nu - \frac{1}{2}f^\mu(\partial^\nu\rho^a_\mu)a^a_\nu \\
&\quad -\frac{1}{2}f^\mu(\partial_\mu\rho^{a\nu})a^a_\nu + \frac{21}{4}f^\mu(\partial^\nu\omega_\nu)f_\mu - 6f^\mu\omega_\mu\partial^\nu f_\nu \\
&\quad -\frac{3}{4}f^\mu(\partial^\nu\omega_\mu)f_\nu - \frac{3}{4}f^\mu(\partial_\mu\omega^\nu)f_\nu + (if_{abc}+d_{abc}) \\
&\quad \times\left(\frac{7}{2}a^{a\mu}(\partial^\nu\rho^a_\nu)a^a_\mu - 4a^{a\mu}\rho^b_\mu\partial^\nu a^c_\nu - \frac{1}{2}a^{a\mu}(\partial^\nu\rho^b_\mu)a^c_\nu\right. \\
&\quad \left.\left.-\frac{1}{2}a^{a\mu}(\partial_\mu\rho^{b\nu})a^c_\nu\right)\right\}\,, \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Gamma_{VVV}}{d\ln\mu^2} &= -\frac{g_V^3}{16\pi^2}\int d^4x\,\{\rho^{a\mu}\omega_\mu\partial^\nu\rho^a_\nu + \frac{1}{2}\rho^{a\mu}\rho^a_\mu\partial^\nu\omega_\nu \\
&\quad +\rho^{a\mu}(\partial^\nu\rho^a_\mu)\omega_\nu + \rho^{a\mu}(\partial_\mu\rho^{a\nu})\omega_\nu + \rho^{a\mu}(\partial_\mu\omega^\nu)\rho^a_\nu \\
&\quad +\frac{3}{4}\omega^\mu\omega_\mu\partial^\nu\omega_\nu + \frac{3}{2}\omega^\mu\omega^\nu\partial_\mu\omega_\nu - (if_{abc}+d_{abc}) \\
&\quad \times\left(\frac{7}{2}\rho^{a\mu}(\partial^\nu\rho^b_\nu)\rho^c_\mu - 4\rho^{a\mu}\rho^b_\mu\partial^\nu\rho^c_\nu - \frac{1}{2}\rho^{a\mu}(\partial^\nu\rho^b_\mu)\rho^c_\nu\right. \\
&\quad \left.\left.-\frac{1}{2}\rho^{a\mu}(\partial_\mu\rho^{b\nu})\rho^c_\nu\right)\right\}\,, \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Gamma_{SSSS}}{d\ln\mu^2} &= -\frac{g_S^4}{\pi^2}\int d^4x\left\{\frac{1}{12}S^4 + \frac{1}{16}S^aS^bS^cS^d\right. \\
&\quad \times[d_{cde}(if_{abe}+d_{abe}) - d_{bde}(if_{aec}+d_{aec}) \\
&\quad \left.+d_{ade}(if_{ebc}+d_{ebc})]\right\}\,, \qquad (40)
\end{aligned}
$$

$$
\Gamma_{SSSP} = \Gamma_{SPPP} = 0\,, \qquad (41)
$$

$$
\begin{aligned}
\frac{d\Gamma_{SSPP}}{d\ln\mu^2} &= -\frac{4g_S^2g_P^2}{\pi^2}\int d^4x\{\frac{1}{12}S^2P^2 \\
&+\frac{1}{16}S^aS^bP^cP^d[d_{cde}(if_{abe}+d_{abe}) \\
&-d_{bde}(if_{aec}+d_{aec})+d_{ade}(if_{ebc}+d_{ebc})]\}\,,
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
\frac{d\Gamma_{PPPP}}{d\ln\mu^2} &= -\frac{4g_P^4}{\pi^2}\int d^4x\{\frac{1}{12}P^4+\frac{1}{16}P^aP^bP^cP^d \\
&\times[d_{cde}(if_{abe}+d_{abe})-d_{bde}(if_{aec}+d_{aec}) \\
&+d_{ade}(if_{ebc}+d_{ebc})]\}\,,
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
\frac{d\Gamma_{AASS}}{d\ln\mu^2} &= -\frac{g_S^2g_A^2}{12\pi^2}\int d^4x\left\{\frac{1}{3}a^{a\mu}a^a_\mu S^bS^b+\frac{1}{4}a^{a\mu}a^b_\mu S^cS^d\left[d_{cde}(if_{abe}+d_{abe})\right.\right. \\
&-d_{bde}(if_{aec}+d_{dec})+d_{ade}(if_{ebc}+d_{ebc})] \\
&\left.+a^{a\mu}f_\mu S^bS^c(if_{abc}+d_{abc})+\frac{1}{2}f^\mu f_\mu S^aS^a\right\}\,,
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
\frac{d\Gamma_{AAPP}}{d\ln\mu^2} &= -\frac{g_P^2g_A^2}{12\pi^2}\int d^4x\left\{\frac{1}{3}a^{a\mu}a^a_\mu P^bP^b+\frac{1}{4}a^{a\mu}a^b_\mu P^cP^d\left[d_{cde}(if_{abe}+d_{abe})\right.\right. \\
&-d_{bde}(if_{aec}+d_{dec})+d_{ade}(if_{ebc}+d_{ebc})] \\
&\left.+a^{a\mu}f_\mu P^bP^c(if_{abc}+d_{abc})+\frac{1}{2}f^\mu f_\mu P^aP^a\right\}\,,
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
\frac{d\Gamma_{VVSS}}{d\ln\mu^2} &= -\frac{g_S^2g_V^2}{12\pi^2}\int d^4x\left\{\frac{1}{3}\rho^{a\mu}\rho^a_\mu S^bS^b+\frac{1}{4}\rho^{a\mu}\rho^b_\mu S^cS^d\left[d_{cde}(if_{abe}+d_{abe})\right.\right. \\
&-d_{bde}(if_{aec}+d_{aec})+d_{ade}(if_{ebc}+d_{ebc})] \\
&\left.+\rho^{a\mu}\omega_\mu S^bS^c(if_{abc}+d_{abc})+\frac{1}{2}\omega^\mu\omega_\mu S^aS^a\right\}\,,
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
\frac{d\Gamma_{VVPP}}{d\ln\mu^2} &= -\frac{g_P^2 g_V^2}{12\pi^2}\int d^4x \left\{ \frac{1}{3}\rho^{a\mu}\rho^a_\mu P^b P^b + \frac{1}{4}\rho^{a\mu}\rho^b_\mu P^c P^d \left[d_{cde}(if_{abe}+d_{abe}) \right.\right. \\
&\left. -d_{bde}(if_{aec}+d_{aec}) + d_{ade}(if_{ebc}+d_{ebc}) \right] \\
&\left. +\rho^{a\mu}\omega_\mu P^b P^c (if_{abc}+d_{abc}) + \frac{1}{2}\omega^\mu\omega_\mu P^a P^a \right\} , \qquad (47)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Gamma_{AAVV}}{d\ln\mu^2} &= \frac{g_A^2 g_V^2}{6\pi^2}\int d^4x (2\delta^{\mu\nu}\delta^{\lambda\sigma} - 7\delta^{\mu\lambda}\delta^{\nu\sigma} - \delta^{\mu\sigma}\delta^{\nu\lambda}) \\
&\times \left\{ \frac{1}{16} a^a_\mu a^b_\nu \rho^c_\lambda \rho^d_\sigma \left[\frac{4}{3}(\delta^{ab}\delta^{cd} - \delta^{bd}\delta^{ac} + \delta^{ad}\delta^{bc}) \right.\right. \\
&\left. + d_{cde}(if_{abe}+d_{abe}) - d_{bde}(if_{aec}+d_{aec}) + d_{ade}(if_{ebc}+d_{ebc}) \right] \\
&+ \frac{1}{8}(a^a_\mu a^b_\nu \rho^c_\lambda \omega_\sigma + a^a_\mu a^b_\nu \rho^c_\lambda \omega_\sigma + a^a_\mu f_\nu \rho^b_\lambda \rho^c_\sigma + f_\mu a^a_\nu \rho^b_\lambda \rho^c_\sigma) \\
&\times (if_{abc}+d_{abc}) + \frac{1}{8}(a^a_\mu a^a_\nu \omega_\lambda \omega_\sigma + a^a_\mu f_\nu \rho^a_\lambda \omega_\sigma + a^a_\mu f_\nu \omega_\lambda \rho^a_\sigma \\
&\left. + f_\mu a^a_\nu \rho^a_\lambda \omega_\sigma + f_\mu a^a_\nu \omega_\lambda \rho^a_\sigma + f_\mu f_\nu \rho^a_\lambda \rho^a_\sigma) + \frac{3}{16} f_\mu f_\nu \omega_\lambda \omega_\sigma \right\} , \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Gamma_{AAAA}}{d\ln\mu^2} &= \frac{g_A^4}{24\pi^2}\int d^4x (2\delta^{\mu\nu}\delta^{\lambda\sigma} - 7\delta^{\mu\lambda}\delta^{\nu\sigma} - \delta^{\mu\sigma}\delta^{\nu\lambda}) \\
&\times \left\{ \frac{1}{16} a^a_\mu a^b_\nu a^c_\lambda a^d_\sigma \left[\frac{4}{3}(\delta^{ab}\delta^{cd} - \delta^{bd}\delta^{ac} + \delta^{ad}\delta^{bc}) \right.\right. \\
&\left. + d_{cde}(if_{abe}+d_{abe}) - d_{bde}(if_{aec}+d_{aec}) + d_{ade}(if_{ebc}+d_{ebc}) \right] \\
&+ \frac{1}{8}(a^a_\mu a^b_\nu a^c_\lambda f_\sigma + a^a_\mu a^b_\nu a^c_\sigma f_\lambda + a^a_\mu f_\nu a^b_\lambda a^c_\sigma + f_\mu a^a_\nu a^b_\lambda a^c_\sigma) \\
&\times (if_{abc}+d_{abc}) + \frac{1}{8}(a^a_\mu a^a_\nu f_\lambda f_\sigma + a^a_\mu f_\nu a^a_\lambda f_\sigma \\
&+ a^a_\mu f_\nu f_\lambda a^a_\sigma + f_\mu a^a_\nu a^a_\lambda f_\sigma + f_\mu a^a_\nu f_\lambda a^a_\sigma + f_\mu f_\nu a^a_\lambda a^a_\sigma) \\
&\left. + \frac{3}{16} f_\mu f_\nu f_\lambda f_\sigma \right\} , \qquad (49)
\end{aligned}
$$

$$
\frac{d\Gamma_{VVVV}}{d\ln\mu^2} = \frac{g_V^4}{24\pi^2}\int d^4x (2\delta^{\mu\nu}\delta^{\lambda\sigma} - 7\delta^{\mu\lambda}\delta^{\nu\sigma} - \delta^{\mu\sigma}\delta^{\nu\lambda})
$$

$$
\begin{aligned}
&\times\Big\{\frac{1}{16}\rho^a_\mu\rho^b_\nu\rho^c_\lambda\rho^d_\sigma[\frac{4}{3}(\delta^{ab}\delta^{cd}-\delta^{bd}\delta^{ac}+\delta^{ad}\delta^{bc})\\
&+d_{cde}(if_{abe}+d_{abe})-d_{bde}(if_{aec}+d_{aec})\\
&+d_{ade}(if_{ebc}+d_{ebc})]+\frac{1}{8}(\rho^a_\mu\rho^b_\nu\rho^c_\lambda\omega_\sigma+\rho^a_\mu\rho^b_\nu\rho^c_\sigma\omega_\lambda\\
&+\rho^a_\mu\omega_\nu\rho^b_\lambda\rho^c_\sigma+\omega_\mu\rho^a_\nu\rho^b_\lambda\rho^c_\sigma)(if_{abc}+d_{abc})\\
&+\frac{1}{8}(\rho^a_\mu\rho^a_\nu\omega_\lambda\omega_\sigma+\rho^a_\mu\omega_\nu\rho^a_\lambda\omega_\sigma+\rho^a_\mu\omega_\nu\omega_\lambda\rho^a_\sigma+\omega_\mu\rho^a_\nu\rho^a_\lambda\omega_\sigma)\\
&+\omega_\mu\rho^a_\nu\omega_\lambda\rho^a_\sigma+\omega_\mu\omega_\nu\rho^a_\lambda\rho^a_\sigma)+\frac{3}{16}\omega_\mu\omega_\nu\omega_\lambda\omega_\sigma\Big\}\ . \qquad (50)
\end{aligned}
$$

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